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# MANY-BODY APPROACHES TO PARTICLE PRODUCTION AT HIGH ENERGIES\*

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## *Introduction*

We shall discuss three methods that have been borrowed from statistical mechanics for the study of multiparticle production at high energies. These are the statistical-thermodynamic model of Hagedorn and Frautschi, the hydrodynamical model of Landau, and the analogy with particle distributions in a fluid developed by Feynman. The first two are direct models for production mechanisms while the third is a guide to viewing the data as suggested by certain models and by statistical mechanics.

## *Statistical-Thermodynamic Model*

Out of Fermi's<sup>1</sup> idea that particle production at high energies is perhaps describable by a few collective variables, e.g., density in phase space and temperature, there have evolved two more sophisticated attempts to explain this phenomenon. These models will be discussed in this and in the following section.

The first questions to answer are what are the independent particles that can be produced, and what is the hadron spectrum? If we wish at some stages to treat the constituents as noninteracting we must introduce in addition to the stable particles all multiparticle excitations, i.e., resonances, and perhaps less restrictively any strong scattering enhancements as independent constituents. We refer to such a general hadronic state as a cluster or fireball. In order to determine the mass spectrum of such fireballs, Hagedorn<sup>2,3</sup> and Frautschi<sup>4</sup> introduced a bootstrap principle. Accordingly, a fireball is composed of an undetermined number of free hadrons contained in a volume  $V_0 \sim (1/m_\pi)^3$ . The bootstrap condition states that each of the constituent hadrons can likewise be considered a fireball. The justification for treating the constituents within each fireball as free was discussed above. Likewise, the confinement of these particles to a finite volume will account for some of their mutual interaction.

Let  $\rho(m)$  be the density of states of mass  $m$ . As a state of mass  $M$  is made up of states of mass  $m$ , the bootstrap condition requires

$$\frac{V_0}{(2\pi)^3} \rho_{\text{out}}(M) = \sum_{n=2}^{\infty} \left( \frac{V_0}{(2\pi)^3} \right)^n \cdot \frac{1}{n!} \int \prod_{i=1}^n dm_i \rho_{\text{in}}(m_i) d^3p_i \delta \left( \sum \sqrt{p_i^2 + m_i^2} - M \right) \delta(\sum \mathbf{p}_i) \quad (1)$$

We assume that the fireballs are made up of at least two constituent particles. Ideally we would like  $\rho_{\text{out}} = \rho_{\text{in}}$  but will settle for

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$$\frac{\rho_{\text{out}}(m)}{\rho_{\text{in}}(m)} \xrightarrow{m \rightarrow \infty} 1 \quad (2)$$

(This condition is referred to as the strong bootstrap. A weaker form of this condition was considered by Hagedorn<sup>2</sup>:  $\log \rho_{\text{out}} / \log \rho_{\text{in}} \rightarrow 1$ .) The solution to this problem is given by<sup>4-6</sup>

$$\rho(m) = cm^{-3} \exp(m/T_0) \quad (3)$$

with  $T_0$  a parameter which may be obtained empirically by looking at the density of existing low mass states. We find that

$$T_0 \sim 160 \text{ MeV} \quad (4)$$

A system with such a fast rising spectrum may be considered in thermodynamic equilibrium for a limited range of temperatures. If we consider a canonical ensemble of such fireballs we find that the partition function

$$Q = \int e^{-m/T} \rho(m) dm \quad (5)$$

can be defined for  $T < T_0$ . Before we turn to the production mechanism for such fireballs and their subsequent decay into observable hadrons, let us summarize the pertinent facts we shall need:

- (i) Hadronic fireballs have a maximum temperature  $T_0$ . Energy tends to be converted towards increasing the particle number rather than the kinetic energy per particle.
- (ii) From Equation 3 we may obtain the probability of finding differing numbers of hadrons in the fireball:

$$P_n = \frac{(\ln 2)^{n-1}}{(n-1)!} \quad (6)$$

This distribution is peaked for low  $n$ , and thus as a fireball decays at each step in the decay chain it will yield roughly  $\bar{n} = 2.4$  hadrons. Likewise the decay proceeds in a sequence where at each step one of the daughter hadrons has most of the mass. We may therefore view the disintegration of the original fireball as proceeding in a number of steps proportional to the initial mass. Thus, the decay multiplicity will be proportional to the fireball mass.<sup>7</sup> (This result is crucially dependent on the exponent of  $m$  in Equation 3 being strictly less than  $-5/2$ .)

- (iii) Combining these ideas, we obtain the decay spectrum from a fireball of mass  $M$  and temperature  $T(M)$

$$dN \sim M \frac{1}{\exp[E/T(M)] \pm 1} d\mathbf{p}, \quad (7)$$

with the  $\pm 1$  corresponding to emitted fermions or bosons. For the subsequent discussion we shall assume that each fireball is at the maximum temperature  $T_0$ , true at high energies.

The major departure of this production model from Fermi's original one is that unlike the latter we do not assume that a high energy collision results in a single fireball but that such a process yields a distribution of such fireballs with varying velocities, or, more conveniently, varying rapidities. We assume that the fireballs

do not carry any transverse momentum. Let  $\eta$  be the rapidity of a particular fireball. Using 7 we see that the decay distribution of this fireball will be

$$E \frac{dN}{dE d\mathbf{p}} = \frac{1}{\pi} \frac{dN}{dy dp_{\perp}^2} = cM(s, \eta) \frac{\mu ch(y - \eta)}{\exp\left[\frac{\mu ch(y - \eta)}{T_0}\right] \pm 1} \quad (8)$$

where  $\nu$  is the rapidity of the observed particle,  $p_{\perp}$  its transverse momentum, and  $\mu = \sqrt{p_{\perp}^2 + m^2}$ .  $M(s, \eta)$  is the mass of the fireball which is a function of its rapidity and the center of mass energy,  $\sqrt{s}$ .

Let  $F(s, \eta, M)$  be the probability of producing a fireball in a high energy collision. The single particle distribution will then be

$$\frac{1}{\pi} \frac{dN}{dy dp_{\perp}^2} = \int G(\eta, s) \frac{\mu ch(y - \eta)}{\exp\left[\frac{\mu ch(y - \eta)}{T_0}\right] \pm 1} d\eta \quad (9)$$

with  $G(\eta, s) = F(s, \eta, M)M(s, \eta)$ . The functions  $F$ ,  $M$ , and  $G$  are not determined by the model and at present are obtained empirically. Barring perverse behavior of the function  $G(\eta, s)$  we obtain two immediate consequences of this model

$$\frac{dN}{dp_{\perp}} \sim e^{-p_{\perp}/T_0} \quad (10)$$

which certainly has the canonical form for such a distribution, and fits the data only roughly (FIGURE 1). The second prediction is on the production distribution as a function of the observed particle mass

$$\frac{dN}{dM} \sim e^{-M/T_0} \quad (11)$$

Again, this is a reasonable representation of the data (FIGURE 2). However, any model that depends mainly on the transverse mass will fit or fail both sets of experiments.

Thus far we have not placed any severe restriction on the function  $G(\eta, s)$ . We will now see if this freedom will permit us to make this model consistent with other models<sup>8-12</sup> and with some general notions we have about particle production. Can the thermodynamic model be made consistent with Feynman scaling? If  $G(\eta, s)$  depends only on  $\eta_0 - \eta$ , where  $\eta_0 = (\ln s/2)$ , we find that the resulting spectrum does indeed scale. If, further,  $G(\eta_0 - \eta)$  is a slowly varying function, then for  $\eta$  small the production spectrum becomes independent of  $y$  and develops a plateau in rapidity. Both statements may be seen from the following:

$$\begin{aligned} \frac{1}{\pi} \frac{dN}{dy dp_{\perp}^2} &= \int \frac{\mu ch(y - \eta)}{\exp\left[\frac{\mu ch(y - \eta)}{T}\right] \pm 1} G(\eta_0 - \eta) d\eta \\ &= \int \frac{\mu ch\eta'}{\exp\left[\frac{\mu ch\eta'}{T}\right] \pm 1} G(\eta_0 - y + \eta') = F(\eta_0 - y) \xrightarrow{|\eta_0 - y| \rightarrow \infty} \text{const} \quad (12) \end{aligned}$$

We can extend the previous discussion to the production of several particles

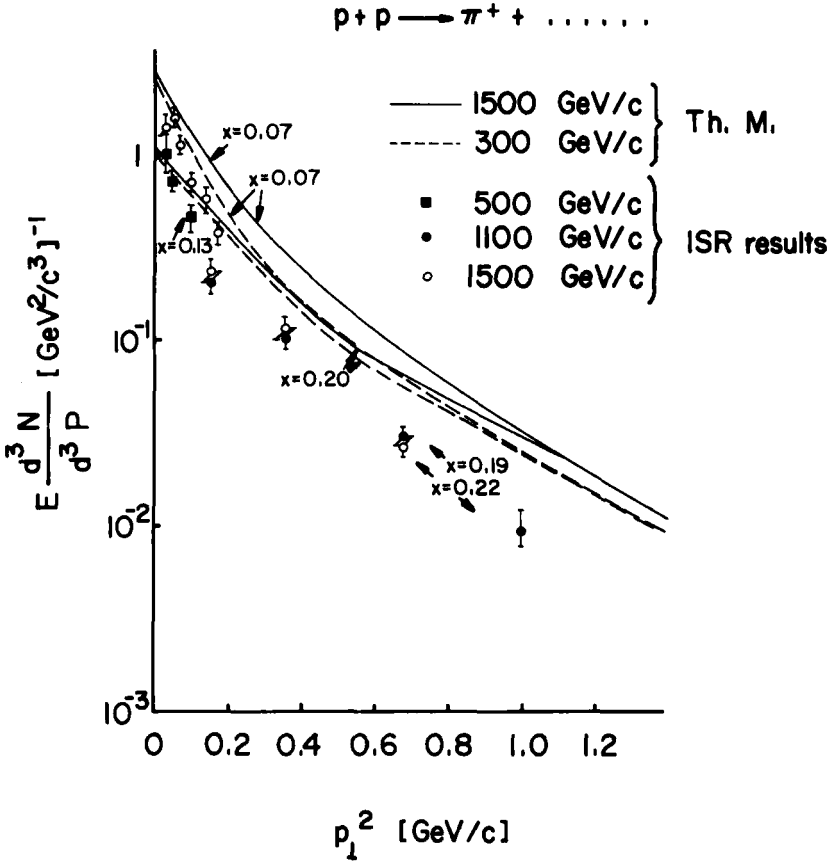


FIGURE 1. Transverse momentum distribution of  $\pi^+$  with a thermodynamic model fit. Details in Reference 11.

and obtain a relation to other models. Multiparticle production can proceed as a decay out of several fireballs or out of one fireball. The decay from multiple fireballs is a product of individual fireball decays; correlations are due to the decay of a single fireball. This  $N$ -fold correlation contribution to the decay spectrum is

$$\begin{aligned}
 \frac{1}{\pi^N} \frac{dC}{dy_1 \dots dp_{1N}^2} &= \int F(s, \eta, M) M^N(s, \eta) \times \prod_{i=1}^N \frac{\mu_i c h(y_i - \eta) d\eta}{\exp\left[\frac{\mu_i c h(y_i - \eta)}{T}\right]} \\
 &= \int G(\eta_0 - \eta) M^{N-1}(s, \eta) \prod_{i=1}^N \frac{\mu_i c h(y_i - \eta) d\eta}{\exp\left[\frac{\mu_i c h(y_i - \eta)}{T}\right]} \pm 1
 \end{aligned} \tag{13}$$

Ranft and Ranft<sup>12</sup> and, in a similar vein, Hamer,<sup>10</sup> consider the following forms for  $M(s, \eta)$ :

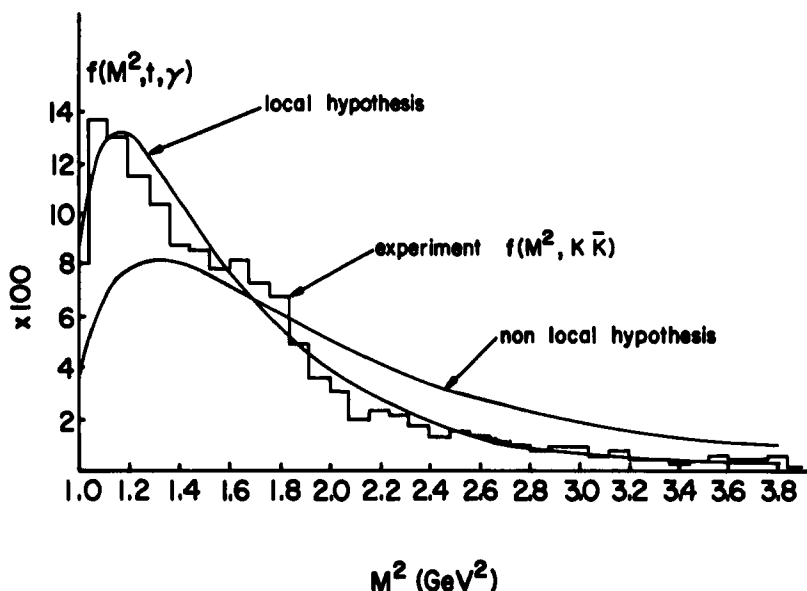


FIGURE 2. Thermodynamic model fit to  $K\bar{K}$  mass spectrum. Details in Reference 8.

$$(a) \quad M(s, \eta) = M_c \left( \frac{e^{\eta_0 - |\eta|}}{M_c} \right)^W \quad 0 \leq W \leq 1$$

$$(b) \quad M(s, \eta) = M_c \ln \left( \frac{e^{\eta_0 - |\eta|}}{M_c} \right)$$

In the two limiting cases of (a) the distribution has a resemblance to that obtained from other models. For  $W = 0$  we obtain short range correlations and  $f_n \sim \ln s$ , as in a multiperipheral model. It has been remarked: that the thermodynamic model may be brought into coincidence with the multiperipheral one by a proper choice of the function  $G(\eta, s)$ . Although, as indicated above, many of the features of the two models are similar when  $W = 0$ , there is one disquieting fact. The two particle rapidity correlation function is

$$C(y_1, y_2) \sim \exp \left\{ - \left[ \frac{m}{T_0} ch |y_1 - y_2| \right] \right\} \quad (14)$$

This correlation decays away much faster than in the multiperipheral correlation function and the coincidence of the two models is somewhat tenuous.

For  $W = 1$  the distribution resembles the one from a diffractive production model<sup>13-15</sup>

$$f_N \sim \sqrt{s}^{N-1} \quad (15)$$

For the intermediate case we obtain

$$f_N \sim (s^{W/2})^{N-1} \quad (16)$$

Case (b) yields

$$f_N \sim (\log s)^N \quad (17)$$

and has no obvious relation with other models.

In summary it may be worthwhile to list the predictions of this model which follow from its first principles and those which come more under the category of empirical data fitting. To the first belong exponential shape of the spectrum (which may have interesting consequences for astrophysics), the transverse momentum cut off and the dependence on the produced particle mass. Longitudinal momentum distributions, correlation functions belong to the second group.

### *Hydrodynamical Model*

The second elaboration on Fermi's model of particle production which we shall discuss is Landau's<sup>16</sup> hydrodynamical model. Some of the predictions of this model have been recently resurrected<sup>17</sup> from hibernation. Because Carruthers<sup>18</sup> discusses this model in more detail, for the sake of completeness, we shall give only a brief summary.

Unlike in the thermodynamic model of the previous section we assume that only one fireball, with a Lorentz contracted volume,

$$V \sim a^3 \sqrt{1 - v^2} \sim a^3 / E_{C.M.} \quad (18)$$

$$a \sim 1/m_\pi$$

is produced. The matter inside heats up without limit and the hadronic matter obeys an ultrarelativistic equation of state

$$p = \epsilon/3 \quad (19)$$

$p$  and  $\epsilon$  are the pressure and energy densities, respectively. Since the number of particles is not fixed, the chemical potential  $\mu$  is equal to 0, implying

$$\epsilon - Ts + p = 0 \quad (20)$$

In the above,  $s$  is the entropy density. After the initial formation of the fireball the hadronic matter is assumed to expand as an ideal fluid. This expansion is taken to be adiabatic. We may calculate the total entropy  $s$  at any stage and assume that the particle number is proportional to  $s$ . Combining Equations 18, 19, and 20 we find

$$N \sim \sqrt{E_{C.M.}} \quad (21)$$

This is certainly as good a fit to the charged multiplicity data as the ones obtained from other models.

In order to obtain more information on the spectrum we must solve the relativistic hydrodynamic equations:

$$\partial_\mu T^{\mu\nu} = 0$$

$$T^{\mu\nu} = (p + \epsilon) U^\mu U^\nu + g^{\mu\nu} p \quad (22)$$

$U^\mu$  is the four velocity. We shall sketch a treatment of this problem which is somewhat between that of Landau and the more rigorous discussion by Milekhin.<sup>19</sup> The initial expansion is assumed to be one dimensional, along the incident direction. We are interested in finding the solution to

$$\begin{aligned}\frac{\partial}{\partial t} T^{00} &= \frac{\partial}{\partial z} T^{0z} \\ \frac{\partial}{\partial t} T^{0z} &= \frac{\partial}{\partial z} T^{zz}\end{aligned}\tag{23}$$

which may be expressed in terms of the rapidity  $y(z, t)$  and the proper time  $\tau(t, z)$ :

$$\begin{aligned}y &= \frac{1}{2} \ln \frac{t+z}{t-z} \\ \tau &= \sqrt{t^2 - z^2}\end{aligned}\tag{24}$$

The entropy density is

$$\ln s = \frac{1}{6} \left[ \sqrt{\left( \ln \frac{\tau}{\Delta} \right)^2 - y^2} - \ln \frac{\tau}{\Delta} \right]\tag{25}$$

with  $\Delta$  being the width of the initial contracted disk. This one dimensional expansion persists until roughly  $\tau \sim a \sim \Delta E_{\text{C.M.}}$ . After that, the expansion changes character and becomes three dimensional, and the entropy ceases to change its dependence on  $y$ . From Equation 25 we obtain, for small  $y$ ,

$$\begin{aligned}\frac{1}{N} \frac{dN}{dy} &= \frac{\exp(-y^2/2L)}{\sqrt{2\pi L}} \\ L &= \ln E_{\text{C.M.}}\end{aligned}\tag{26}$$

It is this expression we wish to confront with experimental data. It clearly does not obey Feynman scaling and this provides an alternative to most present models.

Instead of comparing Equation 26 with data we will consider a more refined version<sup>20</sup> of this model in which Equation 26 is not taken to be the distribution of final particles but, analogous to the discussion of the previous section, it is taken to be the distribution of fireballs all at temperature  $T = m_\pi$ . Thus the  $dN/d\eta$  of Equation 26 becomes the  $G(y, s)$  of Equation 9. A fit to the inclusive distribution of produced  $\pi^+$ 's was made and is shown in FIGURE 3. The fit was made to the large  $y^*$  points, and when extrapolated to smaller  $y^*$ <sup>21</sup> we see that it *does not* reproduce the observed plateau. Due to this and its general lack of having a scaling limit, it appears that this model does not give a correct description of multiparticle production data.

#### *Feynman Fluid Analogy*

This topic is not a model for the mechanism of particle production, but a utilization of the techniques of statistical mechanics, in conjunction with concrete production models, as a way of looking at the experimental data.<sup>22-26</sup> The hope is that these methods will give us a hint as to possible combination or parametrization of data which will lead to simplification.

The idea stems from making a formal analogy between the production distribution of  $n$  particles at rapidities  $y_1, \dots, y_n$ , transverse momenta  $p_{1\perp}, \dots, p_{n\perp}$ , and the distribution of molecules at analogous positions in a container of length  $Y \sim \ln s$  and transverse extent  $\sim \langle p_\perp \rangle$ . If such an identification is correct, then it leads us immediately to such concepts as short range correlations in rapidity and to the idea of a central plateau in rapidity space.



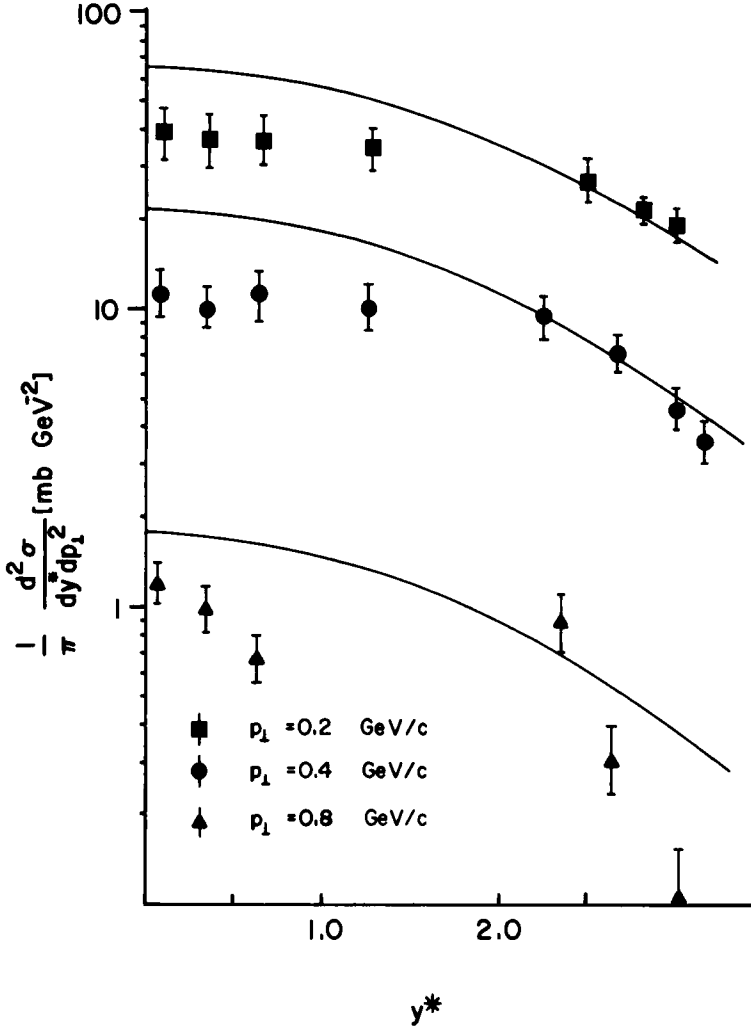


FIGURE 3. Rapidity distribution of  $\pi^+$  at 500 GeV/c. The fit to the large rapidity is from the hydrodynamical model discussed in Reference 20. The small rapidity points are from Reference 21.

Pursuing these hints we are lead to identify the  $N$  particle or  $N$  prong production cross section with the  $N$  particle partition function of the canonical ensemble in statistical mechanics.

$$\sigma_N(s) \leftrightarrow Q_N(Y) \quad (27)$$

Further, one is tempted to form the grand canonical ensemble

$$Q(z, Y) = \sum z^N Q_N(Y) \quad (28)$$

where  $z$  is an arbitrary parameter. Based on models which do possess short range correlation, this function  $Q(z, Y)$  is expected to have certain simple features at high energies, namely,

$$\ln Q(z, Y) \sim p(z)Y + s(z) \quad (29)$$

Whereas each individual  $\sigma_N(s)$  has no obvious simplicity, we expect the combination **28**, hinted at by statistical mechanics, to have the behavior indicated by **29**. This behavior is consistent with experiment (FIGURE 4).

The resulting  $p(z)$ , which is the analog of the pressure, is interesting in that it can be useful in identifying various mechanisms that may be operative in multi-particle production. Different  $z$ 's in Equation **28** weigh varying multiplicities in a differential way; thus we expect to be able to pick out mechanisms that contribute mainly to differing multiplicities. The flat portion of  $p(z)$  plotted in FIGURE 5 could be interpreted as a result of a diffractive mechanism while the rising portion could

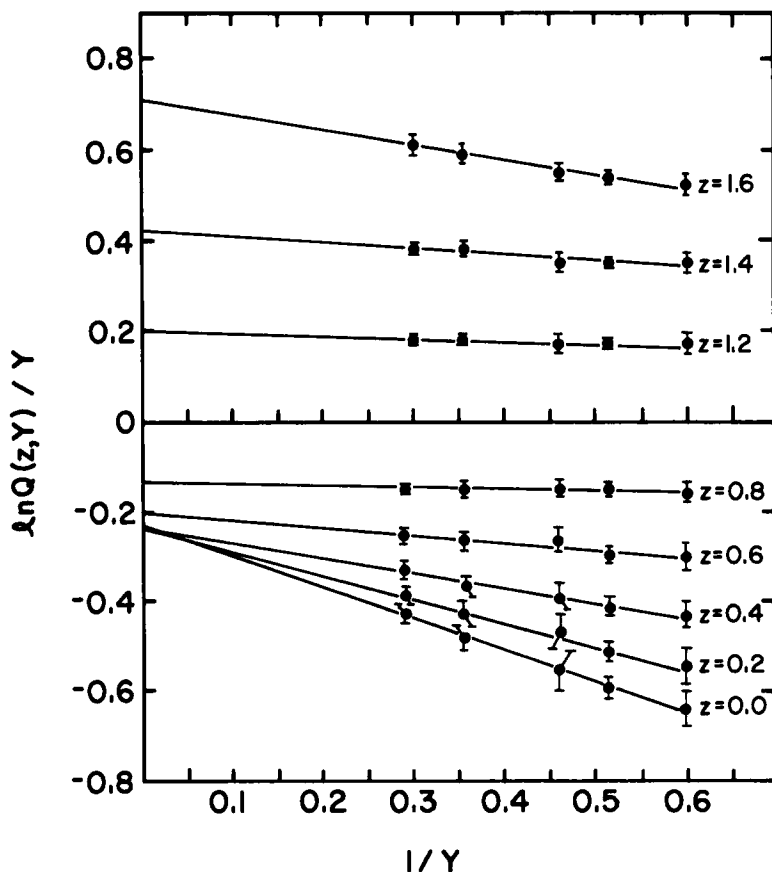


FIGURE 4. Logarithm of the partition function and a best straight line fit. Details in Reference 26.

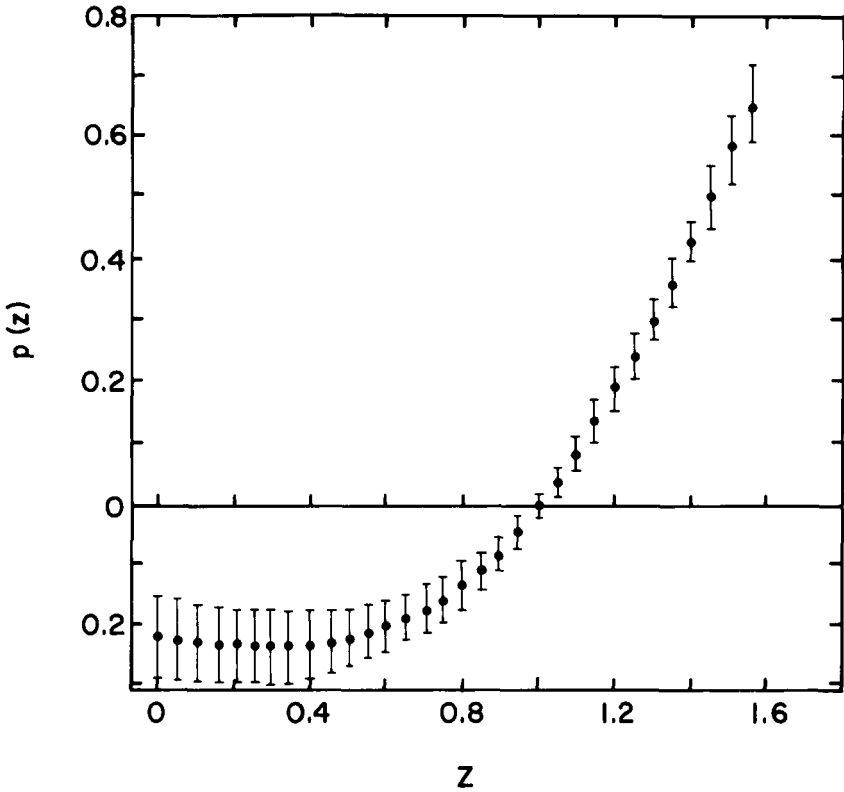


FIGURE 5. Partial pressure due to negative particles. Details in Reference 26.

be due to a multiperipheral production model. Within the context of this analogy one may use relations such as 29 to extrapolate to higher energies and estimate multiplicity distributions there.

Returning to Equation 27 we should obtain the pressure in the canonical ensemble.<sup>27</sup> This leads us to expect that

$$\frac{\partial}{\partial Y} \ln Q_N(Y) = p(N, Y) \quad (30)$$

will be a function of the ratio  $N/Y$  only.

Within the present experimental accuracy this scaling relation appears to be satisfied (FIGURE 6).

Pursuing the hope that the analogy may suggest ways of grouping experimental data so that interesting simplifications occur, we can ask for information from the unintegrated distribution. The fully unintegrated distribution is certainly too cumbersome in the many particle case. The fully integrated prong cross sections may hide interesting features. An immediate technique that suggests itself is to look at the one, two, and so forth, particle distributions in the grand canonical ensemble.<sup>25</sup> Namely, if  $\rho_N(y_1, \dots, y_M)$  is the distribution of  $M$  particles in a final state containing  $N$  particles, one may study

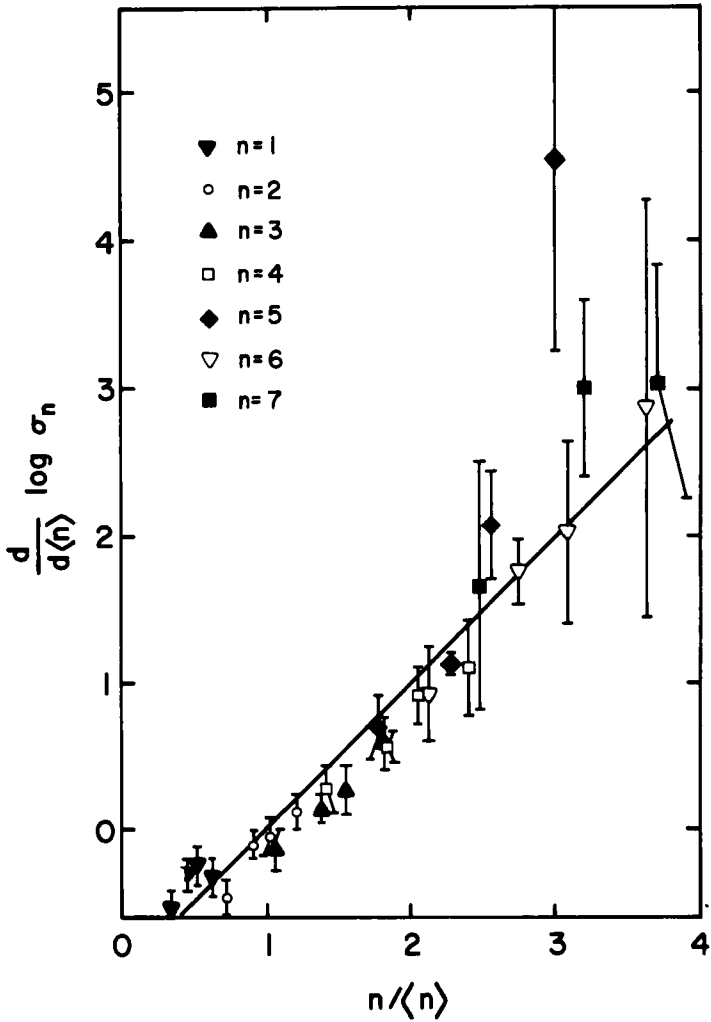


FIGURE 6.  $d/d \langle n \rangle \ln \sigma_n$  vs  $n/\langle n \rangle$  for  $pp \rightarrow n$  negative particles. Details in Reference 27.

$$\rho(y_1, \dots, y, z) = \sum z^N \rho_N(y_1, \dots, y_M) \tag{31}$$

Again the analogy will simplify features of such a distribution.

A more ambitious<sup>28</sup> approach as to how to sample differentially various regions of phase space has been made by Arnold. The crucial assumption (consistent with multiperipheral models) is that the fully unintegrated distribution of  $N$  particles satisfies

$$\frac{d^N \sigma}{dy_2 \cdots dp_{\perp n}} \xrightarrow{|y_i - y_j| \rightarrow \infty} C g^N \tag{32}$$

One then introduces the concept of a short ranged  $N$  body potential

$$\frac{d^N \sigma}{dy_1, \dots, dp_{\perp N}} = C g^N \exp[-U(y_1, p_{1\perp}, \dots, y_N, p_{N\perp})] \quad (33)$$

where  $U$  vanishes whenever all the rapidities separate. Instead of integrating Equation 33 over all phase space in order to obtain  $\sigma_N$ , we introduce a temperature via

$$Q_N(\beta, Y) = C g^N \int d^N \varphi \exp(-\beta U) \quad (34)$$

$Q_N(\beta, Y)$  certainly contains more information about the distribution than does  $\sigma_N$  and is likewise more susceptible to perusal by the human eye. Unfortunately we do not as yet have sufficient experimental data to test the utility of either of these approaches.

The use of this analogy has been fruitful in obtaining an intuitive feeling for many of the features of multiparticle production. However, its foundations rest on both experiment and more direct production models. Not only is it interesting to show the validity of this analogy, but just as interesting will be to see where this analogy does break down, for we know that multiparticle production is not a gas of molecules.

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